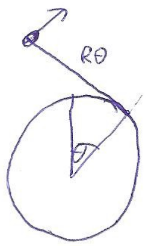
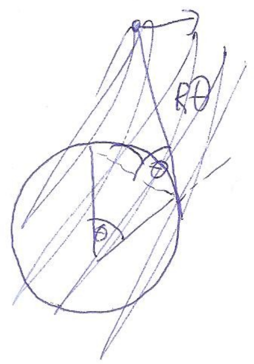
(4)

a) $\mathcal{L} = \frac{1}{2}m(R\Theta)^2\dot{\Theta}^2$





b) $\Rightarrow$ równanie ruchu: $\frac{d}{dt}(\Theta^2\dot{\Theta}) - \dot{\Theta}^2\Theta = 0 \longrightarrow \Theta^2\ddot{\Theta} + \dot{\Theta}^2\Theta = 0$

dla $\Theta \neq 0$: $\frac{d}{dt}(\underset{\uparrow \atop \Theta\frac{d\Theta}{dt}}{\dot{\Theta}\Theta}) = 0 \longrightarrow \frac{d^2}{dt^2}\left(\frac{1}{2}\Theta^2\right) = 0 \Rightarrow \Theta^2 = 2At + B$

war. pocz. $\Theta(0) = 0 \Rightarrow B = 0$

$\mathcal{L}_0 = \frac{1}{2}mv_0^2 \Rightarrow \Theta(0)\dot{\Theta}(0) = v_0/R \Longrightarrow A = \frac{v_0}{R}$

zatem $\boxed{\Theta = \sqrt{2\frac{v_0}{R}t}}$

Energia tak, nie ma zew. źródeł

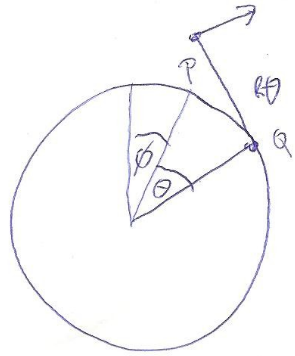
Moment pędu nie, jest siła z pieńka (inaczej byłaby głupia syt. z energią)

c) pieńek: $T_p = \frac{1}{2}MR^2\dot{\phi}^2$

~~P~~ ciężarek: dwa kierunki

$T_c = \frac{1}{2}m\underbrace{R^2\Theta^2}_{\text{rozwinięte}}(\dot{\Theta} + \underset{\uparrow \atop \text{kąt do pionu}}{\dot{\phi}})^2$

$+ \frac{1}{2}mR^2\dot{\phi}^2 \leftarrow$ ruch Q



Zatem

$$L = T_p + T_k = \frac{1}{2}(m+M)R^2\dot{\phi}^2 + \frac{1}{2}mR^2\theta^2(\dot{\theta}+\dot{\phi})^2$$

<u>Równania</u>

$$\frac{\partial L}{\partial \phi} = 0 \Rightarrow \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\phi}}\right) = 0 \; ; \; \underbrace{(m+M)\dot{\phi} + m\theta^2(\dot{\theta}+\dot{\phi}) = 0}_{\text{zasada zachowania momentu pędu}}$$

$$\text{ZZE:} \quad \frac{mv_0^2}{2} = \frac{1}{2}(m+M)R^2\dot{\phi}^2 + \frac{1}{2}mR^2\theta^2(\dot{\theta}+\dot{\phi})^2$$

$\dot{\phi}$ wstawiamy ~~pierwsze do drugiego~~ z pierwszego:

$$\dot{\phi} = \frac{-\dot{\theta}\theta^2}{\theta^2+1+M/m} \; ; \text{ wstawiamy do 2: } ; \; \alpha \equiv 1+\frac{M}{m}, \; \beta \equiv \frac{v_0^2}{R^2}$$

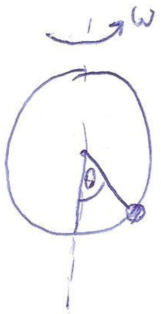
$$\Rightarrow \boxed{\frac{\dot{\theta}^2\theta^2}{\theta^2+\alpha} = \frac{\beta}{\alpha}} \Rightarrow \cancel{\sqrt{\theta^2}} \; \frac{\dot{\theta}\theta}{\sqrt{\theta^2+\alpha}} = \sqrt{\beta/\alpha}$$

całkujemy

$$\Rightarrow \sqrt{\dot{\theta}^2+\alpha} = \sqrt{\beta/\alpha}\, t + C \; \rightarrow \text{kwadratowe oraz } \theta(0) = 0$$

$$\Rightarrow \theta = \sqrt{\frac{\beta}{\alpha}t^2 + 2\sqrt{\beta}\, t}$$

⑤ $d\underline{r} = \hat{r}\,dr + \hat{\theta}\,r\,d\theta + \hat{\phi}\,r\sin\theta\,d\phi$



a) $\underline{v}^2 = (R\dot\theta)^2 + (\omega R\sin\theta)^2$

$T = \frac{1}{2}mv^2 = \frac{1}{2}mR^2\left[\dot\theta^2 + \omega^2\sin^2\theta\right]$

$U = -mgR\cos\theta$

$L = T - U = \frac{1}{2}mR^2\left[\dot\theta^2 + \omega^2\sin^2\theta\right] + mgR\cos\theta$

$= \frac{1}{2}mR^2\dot\theta^2 - V_{eff}$; $V_{eff} = -\frac{1}{2}mR^2\omega^2\sin^2\theta - mgR\cos\theta$

b) Równania E-L

$ml^2\ddot\theta - ml^2\omega^2\sin\theta\cos\theta + mgl\sin\theta = 0$

$\ddot\theta = 0$ gdy $l^2\omega^2\cos\theta = gl \rightarrow \cos\theta = gl/\omega^2 l^2 = g/\omega^2 l$

lub $\sin\theta = 0 \rightarrow \theta = 0$ lub $\theta = \pi$

z pierwszego jest warunek $\omega^2 \geq g/l$; (to samo jest z $\frac{\partial V_{ef}}{\partial\theta} = 0$)

stabilność skoro $ml^2\ddot\theta = -\frac{\partial V_{eff}}{\partial\theta}$; stabilne gdy $\frac{\partial^2 V_{eff}}{\partial\theta^2} > 0$

$\frac{\partial^2 V_{eff}}{\partial\theta^2} = -ml^2\omega^2\cos^2\theta + ml^2\omega^2\sin^2\theta + mgl\cos\theta$

a) $\theta = 0 \rightarrow \frac{\partial^2 V_{ef}}{\partial\theta^2} = -ml^2\omega^2 + mgl = \begin{cases} >0 & \text{gdy } \omega^2 < g/l \\ <0 & \text{gdy } \omega^2 > g/l \end{cases}$

jest stabilne do ω_0 ale dla $\omega > \omega_0$ pojawia się nowy punkt równowagi, jeśliby tam się sprowadzić !

$\theta = \pi/2$ $= -ml^2\omega^2 - mgl < 0$; zawsze niestabilne ostatni (dla $\omega > \sqrt{g/l}$):

$$\frac{\partial^2 U_{ef}}{\partial \theta^2} = -2m\omega^2 l^2 \, g^2/\omega^4 l^2 + m\omega^2 l^2 + mgl \, g/\omega^2 l$$

$$= -mg^2/\omega^2 + m\omega^2 l^2 = \begin{cases} >0 & \text{gdy } \omega^2 > g/l \\ <0 & \text{gdy } \omega^2 < g/l \end{cases}$$

ale jak ię ten punkt jest tylko dla $\omega^2 > g/l$, to jest zawsze stabilny

b) energia nie jest stała, bo wykonywana jest ciągle praca